CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2012 series

9231 FURTHER MATHEMATICS

9231/11 Paper 1, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Qu No	Commentary	Solution	Marks	Part Mark	
1	Re-writes equation	$r\cos\left(\theta - \frac{\pi}{4}\right) = \sqrt{2}$	M1		
	and uses compound angle formula.	$r\left(\frac{\cos\theta}{\sqrt{2}} + \frac{\sin\theta}{\sqrt{2}}\right) = \sqrt{2}$	A1		
	Changes to cartesian.	$\Rightarrow r\cos\theta + r\sin\theta = 2 \Rightarrow x + y = 2$ or $y = 2 - x$.	A1	3	
	Sketches graph.	Straight line at $-\frac{\pi}{4}$ to the initial line.	B1		
		Point (2,0) clearly indicated.	B1	2	**[5]**
2 (i)	Uses formula for mean value.	$\frac{\int_0^4 2x^{\frac{1}{2}}\,\mathrm{d}x}{4}$	M1		
	Integrates	$= \left[\frac{1}{3}x^{\frac{3}{2}}\right]_0^4 = \frac{8}{3}$	M1A1	3	
(ii)	Uses formula for y-coordinate.	$\frac{\frac{1}{2}\int_0^4 4x\,\mathrm{d}x}{\int_0^4 2x^{\frac{1}{2}}\,\mathrm{d}x}$	M1		
	Integrates.	$= \frac{\left[x^2\right]_0^4}{\left[\frac{4}{3}x^{\frac{3}{2}}\right]_0^4} = \frac{16\times 3}{32} = \frac{3}{2}$	M1A1	3	**[6]**
3	Solves AQE.	$m^2 + 4m + 13 = 0 \Rightarrow m = -2 \pm 3\,\mathrm{i}$	M1		
	Finds CF.	CF: $\mathrm{e}^{-2t}(A\cos 3t + B\sin 3t)$	A1		
	Form for PI and differentiates.	PI: $x = pt^2 + qt + r \Rightarrow \dot{x} = 2pt + q \Rightarrow \ddot{x} = 2p$	M1		
	Compares cefficients and solves.	$13p = 26 \Rightarrow p = 2$ $\quad 8p + 13q = 3 \Rightarrow q = -1$ $2p + 4q + 13r = 13 \Rightarrow r = 1$	M1A1		
		GS: $x = \mathrm{e}^{-2t}(A\cos 3t + B\sin 3t) + 2t^2 - t + 1$	A1	6	**[6]**

<table>
<tr><th>Qu No</th><th>Commentary</th><th>Solution</th><th>Marks</th><th>Part Marks</th><th></th></tr>
<tr><td>4</td><td>Verifies given result.

Uses method of differences to sum first series.</td><td>$r(r+1)(r+2)-(r-1)r(r+1)=r(r+1)(r+2-r+1)$
$=3r(r+1)$ (AG)

$\sum_{r=1}^{n} r(r+1)=$
$\frac{1}{3}\{[f(n)-f(n-1)]+[f(n-1)-f(n-2)]+...+[f(1)-f(0)]\}$
$=\frac{1}{3}n(n+1)(n+2)$ (AG) (Award B1 if ‘not hence’.)</td><td>B1

M1

A1</td><td>1

2</td><td></td></tr>
<tr><td></td><td>Subtracts $\sum_{r=1}^{n} r$ to obtain sum of second series.</td><td>$\sum_{r=1}^{n} r^2=\sum_{r=1}^{n} r(r+1)-\sum_{r=1}^{n} r=\frac{n(n+1)(n+2)}{3}-\frac{n(n+1)}{2}$
$=\frac{1}{6}n(n+1)(2n+4-3)=\frac{1}{6}n(n+1)(2n+1)$ (AG)</td><td>M1

A1</td><td>

2</td><td></td></tr>
<tr><td></td><td>Splits series into two series.

Applies sum of squares formula to obtain result.</td><td>$(1^2+2^2+...+n^2)+(2^2+4^2+...+(n-1)^2)=$
$\frac{n(n+1)(2n+1)}{6}+\frac{4\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)n}{6}=...=\frac{1}{2}n^2(n+1)$</td><td>M1

M1A1</td><td>

3</td><td>

[8]</td></tr>
<tr><td>5</td><td>Integrates by parts to obtain reduction formula.</td><td>$\int_0^{\infty} x^n e^{-2x}dx=\left[x^n\frac{-e^{-2x}}{2}\right]_0^{\infty}+\int_0^{\infty} nx^{n-1}\frac{e^{-2x}}{2}dx$
$=\frac{n}{2}I_{n-1}$ (AG)</td><td>M1

A1</td><td>

2</td><td></td></tr>
<tr><td></td><td>(States proposition.)

Proves base case.</td><td>$P_n: I_n=\frac{n!}{2^{n+1}}$

$n=1$ $I_0=\int_0^{\infty} e^{-2x}dx=\left[-\frac{1}{2}e^{-2x}\right]_0^{\infty}=\frac{1}{2}$
$I_1=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{1!}{2^2}$ $\therefore P_1$ true.</td><td>

B1

B1</td><td></td><td></td></tr>
<tr><td></td><td>Shows $P_k\Rightarrow P_{k+1}$.</td><td>$P_k: I_k=\frac{k!}{2^{k+1}}$ for some integer k.
$\therefore I_{k+1}=\frac{k+1}{2}\times\frac{k!}{2^{k+1}}=\frac{(k+1)!}{2^{k+2}}$
$\therefore P_k\Rightarrow P_{k+1}$</td><td>B1
M1A1</td><td></td><td></td></tr>
<tr><td></td><td>States conclusion.</td><td>Hence by PMI P_n is true for all positive integers n.</td><td>A1</td><td>6</td><td>[8]</td></tr>
</table>

Qu No	Commentary	Solution	Marks	Part Mark	
6	Proves initial result.	$(\cos\theta + \mathrm{i}\sin\theta)^4 = \cos 4\theta + \mathrm{i}\sin 4\theta$			
		$= \mathrm{c}^4 + 4\mathrm{c}^3(\mathrm{is}) + 6\mathrm{c}^2(\mathrm{is})^2 + 4\mathrm{c}(\mathrm{is})^3 + (\mathrm{is})^4$	M1		
		$\cos 4\theta = c^4 - 6\mathrm{c}^2(1-\mathrm{c}^2) + (1-\mathrm{c}^2)^2$	M1		
		$= 8\cos^4\theta - 8\cos^2\theta + 1$ (AG)	A1	3	
	Verifies any two cases (B1)	$\cos\frac{4}{7}\pi = -\cos\left(\pi - \frac{4}{7}\pi\right) = -\cos\frac{3}{7}\pi$			
		$\cos\frac{12}{7}\pi = \cos\left(-\frac{2}{7}\pi\right) = -\cos\left(\pi + \frac{2}{7}\pi\right) = -\cos\frac{9}{7}\pi$	B1		
	Verifies remaining two cases (B1)	$\cos\frac{20}{7}\pi = \cos\frac{6}{7}\pi = -\cos\frac{1}{7}\pi = -\cos\frac{15}{7}\pi$ $\cos 4\pi = 1 = -(-1) = -\cos 3\pi$	B1	2	
	Shows roots of equation to be as given.	$8\cos^4\theta - 8\cos^2\theta + 1 = -(4\cos^3\theta - 3\cos\theta)$ $\Rightarrow 8c^4 + 4c^3 - 8c^2 - 3c + 1 = 0$ (*)	M1		
		$\Rightarrow \cos\frac{1}{7}\pi,\ \cos\frac{3}{7}\pi,\ \cos\frac{5}{7}\pi,\ \text{-1}$ are the roots. (AG)	A1	2	
	States sum of roots of equation.	Sum of roots of (*) are $\frac{-4}{8} = -\frac{1}{2}$ $\Rightarrow$ Result (AG)	M1A1	2	[9]
7	Finds asymptotes to *C*.	Vertical asymptote $x = 2$.	B1		
		$y = \lambda x + 1 + \frac{2}{x-2} \Rightarrow y = \lambda x + 1$ is oblique asymptote.	M1A1	3	
	Differentiates and equates to 0.	$y' = \lambda - 2(x-2)^{-2} = 0$ for turning points.	M1A1		
		$\lambda = \frac{2}{(x-2)^2} > 0 \Rightarrow$ no turning points if $\lambda < 0$.	A1	3	
		Or $y' = 0 \Rightarrow \lambda x^2 - 4\lambda x + 4\lambda - 2 = 0$ Uses discriminant to show $8\lambda < 0 \Rightarrow$ no T.P.s.	(M1A1) (A1)		
	Sketch of graph. Deduct 1 mark for poor forms at infinity. Deduct 1 mark if intersections with axes not shown.	Axes and asymptotes. LH branch. RH branch. (Indicating intersections with axes at (0,0) and (3,0).)	B1 B1B1	3	[9]

Qu No	Commentary	Solution	Marks	Part Mark	
8	Differentiates.	$\dot{x} = t^2 - \frac{1}{t} \qquad \dot{y} = 2t^{\frac{1}{2}}$	B1		
	Squares and adds.	$\frac{\mathrm{d}s}{\mathrm{d}t} = \sqrt{\left(t^2 - \frac{1}{t}\right)^2 + 4t} = \sqrt{\left(t^2 + \frac{1}{t}\right)^2}$	M1A1		
	Uses arc length formula.	$s = \int_1^3 \left(t^2 + \frac{1}{t}\right)\mathrm{d}t$	M1		
	Integrates.	$= \left[\frac{t^3}{3} + \ln t\right]_1^3$	A1√		
	Obtains result.	$= 9 + \ln 3 - \frac{1}{3} = \frac{26}{3} + \ln 3 \qquad (= 9.77)$	A1	6	
	Uses surface area formula.	$S = 2\pi \int_1^3 \left(\frac{4}{3}t^{\frac{3}{2}}\left(t^2 + \frac{1}{t}\right)\right)\mathrm{d}t = \frac{8\pi}{3}\int_1^3 \left(t^{\frac{7}{2}} + t^{\frac{1}{2}}\right)\mathrm{d}t$	M1		
	Integrates.	$= \frac{8\pi}{3}\left[\frac{2}{9}t^{\frac{9}{2}} + \frac{2}{3}t^{\frac{3}{2}}\right]_1^3$	A1		
	Inserts limits.	$= \frac{8\pi}{3}\left\{\left[18\sqrt{3} + 2\sqrt{3}\right] - \left[\frac{2}{9} + \frac{2}{3}\right]\right\}$	M1		
	Obtains result.	$= \pi\left(\frac{160\sqrt{3}}{3} - \frac{64}{27}\right) \qquad (= 283 \text{ or } = 90.0\pi)$			
			A1	4	[10]

Qu No	Commentary	Solution	Marks	Part Mark	Total
9	Finds vector normal to *Π*.	$\mathbf{n}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & -2 & 2\\ 3 & 1 & -2\end{vmatrix}=2\mathbf{i}+8\mathbf{j}+7\mathbf{k}$	M1A1		
	Dot product of this with general point on l_1.	$\begin{pmatrix}3+8t\\ 6+5t\\ 12-8t\end{pmatrix}\cdot\begin{pmatrix}2\\ 8\\ 7\end{pmatrix}=138$ or $\begin{pmatrix}8\\ 5\\ -8\end{pmatrix}\cdot\begin{pmatrix}2\\ 8\\ 7\end{pmatrix}=0$	A1√		
	Deduces result.	Independent of t ⇒ parallel, or ⇒ parallel.	A1	4	
	Cartesian equation of *Π*.	Π: $2x+8y+7z=21$	B1		
	Substitutes general point of l_2.	Sub. $x=5+2s$, $y=-4-s$, $z=7+s$	M1		
	Finds value of parameter.	$\Rightarrow s=-2$	A1		
	Finds p.v. of intersection.	and line meets *Π* at point with p.v. $\mathbf{i}-2\mathbf{j}+5\mathbf{k}$	A1	4	
		Take (9,11,2) as *A*, (3,6,12) as *B* and let *C* be foot of perpendicular from A to *l*.			
	Finds distance from point to known point on *l*.	$AB=\sqrt{6^2}+5^2+10^2=\sqrt{161}$	B1		
	Finds distance along *l* from known point to foot of perpendicular from given point to *l*.	$BC=\frac{1}{\sqrt{(6^2\ 5^2+8^2)(6i+5j-10k)(8i+5j-8k)}}=\frac{153}{\sqrt{153}}=\sqrt{153}$	M1 A1	4	[12]
	F.t. on non-hypotenuse side (must be real).	$AC=\sqrt{161-153=\sqrt{8\text{ or }2\sqrt{2}}}$ (= 2.83)	A1√		
		Alternatively:			
	Writes a set of three equations in three unknowns for the intersection of *l* with *Π*.	$5+2s=2+\lambda+3\mu$ $-4-s=3-2\lambda+\mu$ $7+s=-1+2\lambda-2\mu$	(B1)		
	Solves the set of equations.	$\Rightarrow s=-2$, $\Rightarrow \lambda=2$, $\mu=-1$	(M1A1)		
	Finds p.v. of intersection.	and line meets *Π* at point with p.v. $\mathbf{i}-2\mathbf{j}+5\mathbf{k}$	(A1)		

Qu No	Commentary	Solution	Marks	Part Mark	
9	Finds vector $\overrightarrow{BA}$.	$\overrightarrow{BA} = 6\mathbf{i} + 5\mathbf{j} - 10\mathbf{j}$	B1		
		$\left\lvert \frac{1}{\sqrt{8^2+5^2+8}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 5 & -10 \\ 8 & 5 & -8 \end{vmatrix} \right\rvert$	M1A1		
		$= \sqrt{\frac{1224}{153}} = \sqrt{8}$ or $2\sqrt{2}$ $(= 2.83)$	A1	(4)	
		Alternatively: (A)			
	Finds distance from point to known point on l.	Take (9,11,2) as A, (3,6,12) as B and let C be foot of perpendicular from A to l.	(B1)		
		$AB = \sqrt{6^2+5^2+10^2} = \sqrt{161}$			
	Finds distance along l from known point to foot of perpendicular from given point to l.	$BC = \frac{1}{\sqrt{8^2+5^2+8^2}}(6\mathbf{i}+5\mathbf{j}-10\mathbf{k})(8\mathbf{i}+5\mathbf{j}-8\mathbf{k})$	(M1)		
		$= \frac{153}{\sqrt{153}} = \sqrt{153}$	(A1)		
	F.t. on non-hypotenuse side (must be real).	$AC = \sqrt{161-153} = \sqrt{8}$ or $2\sqrt{2}$ $(= 2.83)$	(A1√)	(4)	
		(B)			
	Finds vector . $\overrightarrow{AC}$	$\overrightarrow{AC} = \begin{pmatrix} 8t-6 \\ 5t-5 \\ 10-8t \end{pmatrix}$	(B1)		
	Uses $\overrightarrow{AC}$ perpendicular to l to find t.	$\begin{pmatrix} 8t-6 \\ 5t-5 \\ 10-8t \end{pmatrix} . \begin{pmatrix} 8 \\ 5 \\ -8 \end{pmatrix} = 0 \Rightarrow t = 1$	(M1A1)		
	Finds length AC.	$AC = \sqrt{2^2+0^2+2^2} = \sqrt{8}$	(A1)	(4)	[12]

Qu No	Commentary	Solution	Marks	Part Mark	Total
10	States eigenvalues.	Eigenvalues are 1, 2, 3.	B1	1	
	Finds eigenvectors.	$\lambda = 1 \quad \mathbf{e}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -16 \\ 0 & 1 & 3 \end{vmatrix} = \begin{pmatrix} 28 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1A1		
		$\lambda = 2 \quad \mathbf{e}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 4 & -16 \\ 0 & 0 & 1 \end{vmatrix} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$	A1		
		$\lambda = 3 \quad \mathbf{e}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 & -16 \\ 0 & -1 & 3 \end{vmatrix} = \begin{pmatrix} -4 \\ 6 \\ 2 \end{pmatrix} \sim \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$	A1	4	
	States **P** and **D**.	$\mathbf{P} = \begin{pmatrix} 1 & 4 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 3^n \end{pmatrix}$	B1√ B1√	2	
	Finds inverse of **P**.	Det $\mathbf{P} = 1 \Rightarrow$ Adj $\mathbf{P} = \mathbf{P}^{-1} = \begin{pmatrix} 1 & -4 & 14 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$	M1A1		
	Finds $\mathbf{A}^n$.	$\mathbf{A}^n = \mathbf{PDP}^{-1}$ $= \mathbf{P}\begin{pmatrix} 1 & -4 & 14 \\ 0 & 2^n & -3.2^n \\ 0 & 0 & 3^n \end{pmatrix}$ or	M1A1		
		$\begin{pmatrix} 1 & 4.2^n & -2.3^n \\ 0 & 2^n & 3^{n+1} \\ 0 & 0 & 3^n \end{pmatrix}\mathbf{P}$	A1	5	
		$= \begin{pmatrix} 1 & [-4 + 4.2^n] & [14 - 12.2^n - 2.3^n] \\ 0 & 2^n & [-3.2^n + 3^{n+1}] \\ 0 & 0 & 3^n \end{pmatrix}$			
	States required limit.	$3^{-n}\mathbf{A}^n \to \begin{pmatrix} 0 & 0 & -2 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ as $n \to \infty$.	B1√	1	[13]

Qu No	Commentary	Solution	Marks	Part Mark	Total
11	**EITHER** Substitute α into equation. Multiply by α^n. Obtain result.	α is a root $\Rightarrow \alpha^4 - 3\alpha^2 + 5\alpha - 2 = 0$ $\Rightarrow \alpha^{n+4} - 3\alpha^{n+2} + 5\alpha^{n+1} - 2\alpha^n = 0$ Repeat for β, γ, δ and sum $\Rightarrow S_{n+4} - 3S_{n+2} + 5S_{n+1} - 2S_n = 0$ (AG)	M1 A1	2	
(i)	Uses $\sum\alpha^2 = \left(\sum\alpha\right)^2 - 2\sum\alpha\beta$ Finds S_4 from formula.	$S_2 = 0 - 2\times(-3) = 6$ $S_4 = 3\times 6 - 5\times 0 + 2\times 4 = 26$	B1 M1A1	3	
(ii)	$S_{-1} = \dfrac{\sum\alpha\beta\gamma}{\alpha\beta\gamma\delta}$ Finds S_3 from formula. Finds S_5 from formula.	$S_{-1} = \dfrac{-5}{-2} = \dfrac{5}{2}$ $S_3 = 3\times 0 - 5\times 4 + 2\times\dfrac{5}{2} = -15$ $S_5 = 3\times(-15) - 5\times 6 + 2\times 0 = -75$	M1A1 M1A1 M1A1	6	
		$\sum\alpha^2\beta^3 = S_2S_3 - S_5$ $= 6\times(-15) - (-75) = -15$	M1 M1A1	3	[14]

Qu No	Commentary	Solution	Marks	Part Mark	
11	**OR**				
(i)	Reduces **M** to echelon form.	$\begin{pmatrix} 2 & 1 & -1 & 4 \\ 3 & 4 & 6 & 1 \\ -1 & 2 & 8 & -7 \end{pmatrix} \to \dots \to \begin{pmatrix} 2 & 1 & -1 & 4 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	M1A1		
	Uses dimension theorem.	Dim(**M**) = 4 – 2 = 2	A1		
(ii)	States basis for R.	Basis for R is $\left\{ \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \right\}$. (OE)	B1		
	Finds cartesian equation for R.	$x = 2\lambda + \mu$ $y = 3\lambda + 4\mu \quad \Rightarrow 2x - y + z = 0$ $z = -\lambda + 2\mu$	M1A1		
(iii)	Finds basis for null space.	$2x + y - z + 4t = 0$ $y + 3z - 2t = 0$ $t = \lambda$ and $z = \mu$ $\Rightarrow y = 2\lambda - 3\mu$ and $x = -3\lambda + 2\mu$	M1		
		$\Rightarrow$ Basis of null space is $\left\{ \begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 1 \\ 0 \end{pmatrix} \right\}$ (OE)	A1A1	9	
	Evaluates k.	$2 \times 8 - 7 + k = 0 \Rightarrow k = -9$	B1		
	Finds a particular solution.	$5\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \\ -9 \end{pmatrix}$ (OE) (via equations)	M1A1		
	Finds general solution.	$\mathbf{x} = \begin{pmatrix} 5 \\ -2 \\ 0 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ -3 \\ 1 \\ 0 \end{pmatrix}$.	M1A1	5	**[14]**